Kearney Air Quality: Dosimetry

The EPA RadNet station pumps air through a filter nominally for 72 hours, or 96 hours. As air flows through the filter nominally at 60 m^3/hour, the filter collects dust. As the dust is collected, radioactive material is captured by the filter at a rate, $\mathcal{R}$, measured in pCi/hour even as the radioactive material is disappearing do to nuclear transmutations.
After the pump is turned off the activity on the filter begins a monotonic decline toward zero. The detector provided is calibrated so that we are easily able to determine the activity (both α, and β) on the filter in units of pCi.
Your objective is to collect and analyze data to determine the dose rate in pCi/m^3 for Kearney's air.

We begin with the observation that radioactive isotopes disappear so that their number $\mathcal{N}$ as a function of time is: $\mathcal{N}_t = \mathcal{N}_0 e^{-\lambda t}$. This leads to the notion of activity $\mathcal{A}$ as defined as the time derivative of the aforementioned function.

1. Show that the time derivative of $\mathcal{A}$ produces equation(1), then solve that differential equation.

$$\frac{d\mathcal{A}}{dt} = -\lambda \mathcal{A} \qquad (1)$$

Solution:

$$\mathcal{A} = (\mathcal{A}_{\text{stop}})\, e^{-\lambda t}$$

Collect data to determine $\mathcal{A}_{\text{stop}}$, and λ for separately α, and β activities.

Use the solution to equation(1) and Table 1 data to determine $\mathcal{A}_{\text{stop}}$, and λ.

(t − t_{stop}) (hr)	0.25	0.75	1.5	2.1	2.75	3.5	4.3
α-activity (pCi)	1125	633	267	134	63	27	11
β-activity (pCi)	7558	4317	1864	952	460	198	81

Table 1: Values to be used to determine solution to equation (1).

2. Solve the differential equation:

$$\frac{d\mathcal{A}}{dt} = \mathcal{R} - \lambda\mathcal{A} \tag{2}$$

Solution:

$$\mathcal{A} = \frac{\mathcal{R}}{\lambda}\left(1 - e^{-\lambda t}\right)$$

Solve for $\mathcal{R}$ in terms of $\mathcal{A}_{\text{stop}}$, t_{stop}, and λ.

$$\mathcal{R} = \frac{\lambda\mathcal{A}_{\text{stop}}}{1 - e^{-\lambda t_{\text{stop}}}} \approx \lambda\mathcal{A}_{\text{stop}} \text{ , for large values of } \lambda t_{\text{stop}}$$

Determine $\mathcal{R}$ by inserting your values of $\mathcal{A}_{\text{stop}}$, and λ, and $\text{t}_{\text{stop}} = 72.73$ hours into your solution to equation(2).

3. Use your calculated value of $\mathcal{R}$ and the air flow rate (vtbd[1]) to calculate the dose rate for Kearney's air.

4. Why do we stop here. Why don't we calculate the rad (and/or sievert) equivalent dose for this example?

[1]Value To Be Determined

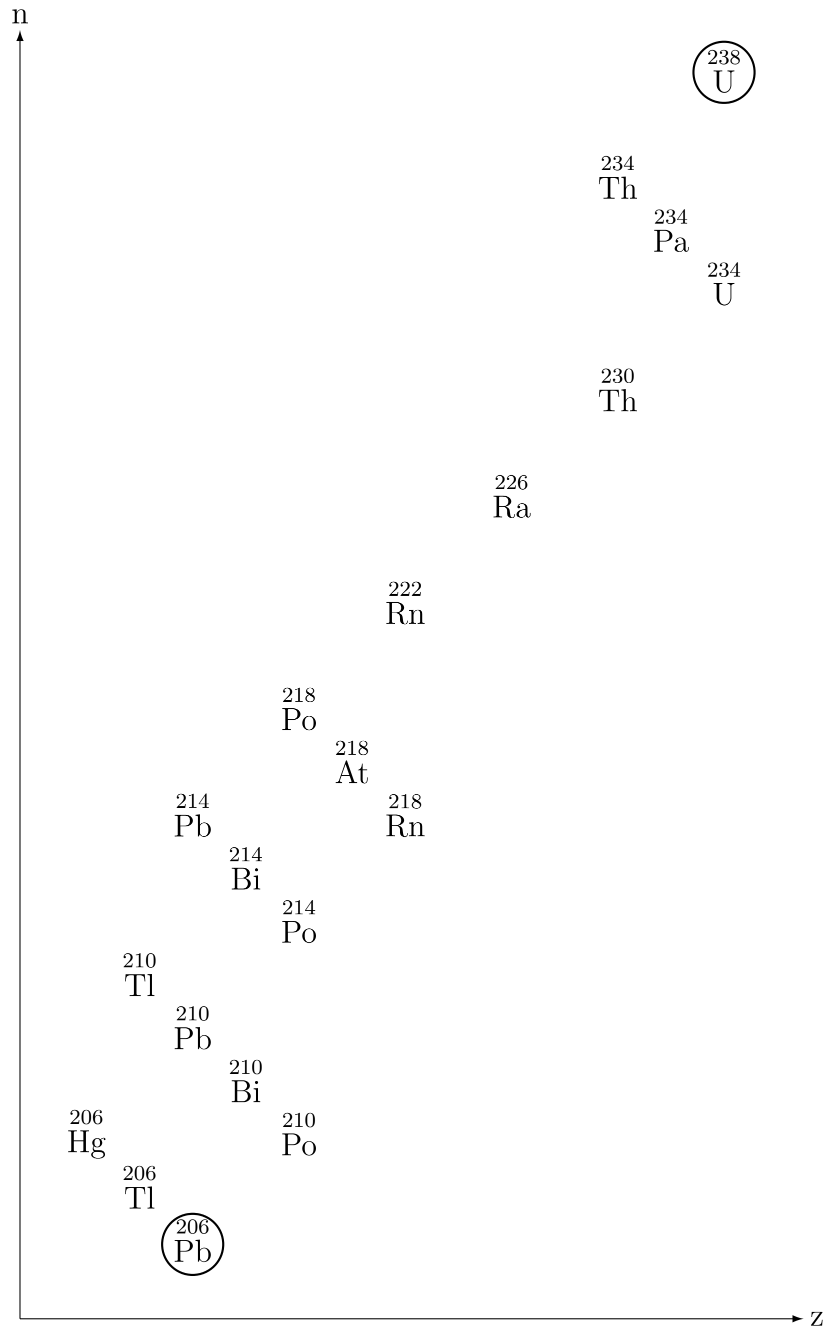

Figure 1: ${}_{92}U^{238} \rightarrow {}_{82}Pb^{206} + N_\alpha({}_2He^4) + N_\beta({}_{-1}e^0) + N_{\bar{\nu}}(\bar{\nu})$ + Energy

1 Radon Daughter Isotopes

Most ($> 99\%$) Uranium is ${}_{92}\mathrm{U}^{238}$, an unstable isotope that transmutes (with a 4.86×10^9 year half-life) by spontaneous nuclear fission into ${}_{2}\mathrm{He}^{4} + {}_{90}\mathrm{Th}^{234}$. This is followed by the transmutation sequence:

$${}_{90}\mathrm{Th}^{234} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{91}\mathrm{Pa}^{234} \qquad \mathrm{t}_{1/2} = 24.1 \text{ days}$$
$${}_{91}\mathrm{Pa}^{234} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{92}\mathrm{U}^{234} \qquad \mathrm{t}_{1/2} = 6.75 \text{ hours}$$
$${}_{92}\mathrm{U}^{234} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{90}\mathrm{Th}^{230} \qquad \mathrm{t}_{1/2} = 2.54 \times 10^{5} \text{ years}$$
$${}_{90}\mathrm{Th}^{230} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{88}\mathrm{Ra}^{226} \qquad \mathrm{t}_{1/2} = 8.0 \times 10^{4} \text{ years}$$

$${}_{88}\mathrm{Ra}^{226} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{86}\mathrm{Rn}^{222} \qquad \mathrm{t}_{1/2} = 1.60 \times 10^{3} \text{ years}$$

$${}_{86}\mathrm{Rn}^{222} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{84}\mathrm{Po}^{218} \qquad \mathrm{t}_{1/2} = 3.82 \text{ days}$$

$\mathrm{t}_{1/2} = 3.05$ minutes

$${}_{84}\mathrm{Po}^{218} \xrightarrow{99+\%}\ {}_{2}\mathrm{He}^{4}\ +\ {}_{82}\mathrm{Pb}^{214} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{83}\mathrm{Bi}^{214} \qquad \mathrm{t}_{1/2} = 26.8 \text{ minutes}$$
$${}_{84}\mathrm{Po}^{218} \xrightarrow{0.018\%}\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{85}\mathrm{At}^{218} \xrightarrow{99+\%}\ {}_{2}\mathrm{He}^{4}\ +\ {}_{83}\mathrm{Bi}^{214} \qquad \mathrm{t}_{1/2} \approx 2 \text{ seconds}$$

$\mathrm{t}_{1/2} \approx 2$ seconds

$${}_{85}\mathrm{At}^{218} \xrightarrow{0.1\%}\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{86}\mathrm{Rn}^{218} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{84}\mathrm{Po}^{214} \qquad \mathrm{t}_{1/2} = 35 \text{ milliseconds}$$

$\mathrm{t}_{1/2} = 19.7$ minutes

$${}_{83}\mathrm{Bi}^{214} \xrightarrow{99+\%}\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{84}\mathrm{Po}^{214} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{82}\mathrm{Pb}^{210} \qquad \mathrm{t}_{1/2} = 164\ \mu\mathrm{s}$$
$${}_{83}\mathrm{Bi}^{214} \xrightarrow{0.021\%}\ {}_{2}\mathrm{He}^{4}\ +\ {}_{81}\mathrm{Tl}^{210} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{82}\mathrm{Pb}^{210} \qquad {}_{1/2} = 1.30 \text{ m}$$

$\mathrm{t}_{1/2} = 22.3$ years

$${}_{82}\mathrm{Pb}^{210} \xrightarrow{99+\%}\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{83}\mathrm{Bi}^{210} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{84}\mathrm{Po}^{210} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{82}\mathrm{Pb}^{206}$$
$${}_{82}\mathrm{Pb}^{210} \xrightarrow{99+\%}\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{83}\mathrm{Bi}^{210} \rightarrow\ {}_{2}\mathrm{He}^{4}\ +\ {}_{81}\mathrm{Tl}^{206} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{82}\mathrm{Pb}^{206}$$
$${}_{82}\mathrm{Pb}^{210} \xrightarrow{1.7\times10^{-6}\%} {}_{2}\mathrm{He}^{4}\ +\ {}_{80}\mathrm{Hg}^{206} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{81}\mathrm{Tl}^{206} \rightarrow\ {}_{-1}\mathrm{e}^{0}\ +\ {}_{82}\mathrm{Pb}^{206}$$

Radium is present in every natural geological location. The filters capture dust particles containing Radium. No isotopes of radium are stable and Ra-226 is the longest lived of those isotopes. Therefore, Radium acts as a reservoir for Radon-222, a noble gas, and its daughter isotopes that constitute a branching chain of transmutations that generate several α, and β particles.

$$<t>=\frac{\int_0^\infty dt \;\; te^{-\lambda t}}{\int_0^\infty dt \;\; e^{-\lambda t}}=\frac{\int_0^\infty d(\lambda t) \;\; (\lambda t)e^{-\lambda t}}{\lambda\int_0^\infty d(\lambda t) \;\; e^{-\lambda t}}=\frac{\int_0^\infty dx \;\; xe^{-x}}{\lambda\int_0^\infty dx \;\; e^{-x}} \tag{3}$$

$$\int_0^\infty dx \;\; xe^{-x}=-xe^{-x}\Big|_0^\infty+\int_0^\infty dx \;\; e^{-x}=\int_0^\infty dx \;\; e^{-x} \tag{4}$$

$$<t>=\lambda^{-1} \text{ , where , } \lambda=\frac{\ln 2}{t_{1/2}} \therefore t_{1/2}=\ln 2<t>\approx 0.693<t> \tag{5}$$

$$<t>=\tau \tag{6}$$