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Use of the "Welch" scattering apparatus

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At that time, the jogger's y coordinate is just

$$y = v \sin \theta t = vL \sin \theta / (V - v \cos \theta).$$

Squaring and rearranging terms

$$v^2(y^2 + L^2)\cos^2 \theta - 2y^2 Vv \cos \theta + (V^2y^2 - v^2L^2) = 0. \quad (5)$$

For real $\cos \theta$, $b^2 - 4ac \geqslant 0$,

$$4y^4v^2V^2 \geqslant 4v^2(y^2 + L^2)(V^2y^2 - v^2L^2),$$

simplifying to

$$y \leqslant vL / \sqrt{V^2 - v^2}.$$

Substitute back into Eq. (5), we get

$$\cos \theta_{opt} = v/V.$$

V. DISCUSSION

The noncalculus technique of finding extreme values for the above class of problems may be summarized as follows:

(a) Write out the equation for the system in the form $a \sin \theta + b \cos \theta + c = 0$, where a, b, and c are functions of the variable of interest.

(b) Change the linear form into a quadratic equation of either sine or cosine (or tangent).

(c) Impose the condition $b^2 - 4ac \geqslant 0$ for a real angle to obtain the extreme value for the variable concerned.

(d) Put $\sin \theta = -b/2a$ (or cosine/tangent whichever is chosen to form the quadratic equation) to obtain the condition for optimization. If the problem requires only the finding of such a condition, the solution obtained by this method is sometimes even quicker than that obtained by following the standard procedure of using differential calculus.

Of course, this approach will not work for every physical problem; however, it does seem to be useful in a number of cases, just to name a few: (1) maximum range of a parabolic projectile,[1] (2) maximum range of an elliptic projectile,[5] (3) some special cases of Fermat's principle,[6] and (4) smallest electric field required for breakdown in high-frequency gas discharge.[7] Also, there are limited cases where the technique $b^2 - 4ac \geqslant 0$ may be used with the starting equation containing no sine or cosine functions. They include: (5) derivation of Archimedes' principle,[1] (6) minimizing of the distance between the object and its real image for a convex lens,[8] (7) resistance matching for a maximum power output in a simple circuit,[8] (8) shortest distance between two travelers moving in mutually perpendicular directions,[9] and (9) optimum reservoir temperature for a finite heat source and a Carnot engine.[1]

The three illustrations selected in the present paper were chosen as ones most interesting, easily applicable, recognizable, and understandable to students in a noncalculus course.

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Use of the "Welch" scattering apparatus

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Atomic or nuclear collisions can be modeled using the "Welch" scattering apparatus. The student obtains and analyzes "macroscopic" data from a scattering experiment in order to ascertain "microscopic" properties (orientation and shape) of a hidden target. The procedure described emphasizes the analysis of data that are analogous to the data obtained in atomic or nuclear collision studies. The student finds that the position of a cluster of scattered projectiles indicates the orientation of the scattering surface while the density of the cluster indicates the shape (radius of curvature) of the scattering surface.

I. INTRODUCTION

In 1961 D. J. Prowse wrote,[1] "The Welch scattering apparatus[2]...is designed to acquaint the student with some of the mechanics of particle scattering as applied in atomic and nuclear physics by the performance of an analogous experiment in which small ball bearings are scattered in two dimensions from a hardened steel [now made of Lucite] cylinder." His note was, "written to point out that it is possible to carry the analogy with nuclear physics much further by introducing the concept of a differential cross section which can actually be measured by the student in this particular case."

The Welch apparatus comes with instructions that include a detailed analysis of a steel ball bearing colliding with a Lucite target of circular cross section. Concerning the use of targets with elliptical cross section in the apparatus, the instructions include the statement, "The purpose of the elliptical targets is to acquaint the user with some of the mechanisms of scattering of atomic particles by using analogous objects of large dimensions. ...However, the analysis of the data is more complex." The reader is then referred to a laboratory manual[3] for detailed analysis of a steel ball bearing colliding with a Lucite target of elliptical cross section. These two treatments for the use of the Welch apparatus have at least one thing in common: they miss the point entirely. When has an atomic or nuclear physicist ever known the impact parameter for a collision?

A physicist works out the dynamics of a collision using microscopic parameters that can never be measured in order to understand the statistics for the parameters that can be measured. Using this observation, in the spirit of Prowse's article, this article will set out the dynamics for the collisions in a form that will allow the shape and orientation of a hidden target to be determined from data that are analogous to the data obtained from real atomic and nuclear physics experiments.

II. THEORY

Following any good modern physics text[4] the differential scattering cross section for a circular target as shown in Fig. 1 is easy to find. Let R_1 be the radius of the target plus the radius of the projectile. Assuming the collisions to be elastic we see that $\alpha = 2\theta$. Simple geometry then gives the impact parameter

$$b = R_1 \sin(\theta) = R_1 \sin(\alpha/2). \quad (1)$$

If I equals the number of projectiles per inch fired at the target then I times $d(b)$ will be the number of projectiles scattered into an angular space $d\alpha$ given by the relationship

$$Id(b) = I(R_1/2) \cos(\alpha/2) d\alpha = N_W, \quad (2)$$

where N_W is the number of projectiles scattered into the "detector window" of angular width $d\alpha$. The differential scattering cross section $d\sigma$ can be taken as

$$d\sigma = N_W/I = (R_1/2) \cos(\alpha/2) d\alpha \,. \quad (3)$$

Again, using simple geometry in Fig. 2 gives $ds = R_2\, d\alpha$, where ds is the width of the rectangular hole in a mask used to simulate the sensitive part of a detector and R_2 is the radius of the scattering apparatus (assumed to be much larger than the size of the target). Solving for R_1 gives the experimentally determined quantity of interest:

$$R_1 = 2N_W R_2/I \cos(\alpha/2) ds. \quad (4)$$

All of the quantities on the right-hand side of this equation are determined with no knowledge of the target's properties. A graph of R_1 vs α for the case of a circular target results in a straight line of zero slope as the target has a radius that is constant at R_1. Equation (2) samples a small

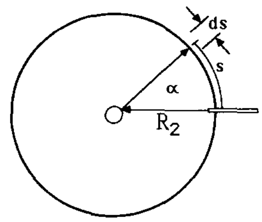

Fig. 2. Projectiles scatter from the target into a detector of widths ds.

part of the target if $d\alpha$ is small. If this is true then R_1 of Eq. (4) represents the radius of curvature of that part of the target.

The corresponding equations for an arbitrarily shaped target are not so transparent. The perimeter of an arbitrary shape shown in Fig. 3 may be defined by a function $R(\theta)$, where $R(\theta)$ usually represents the distance from an origin to the perimeter. For this discussion however, $R(\theta)$ will represent the radius of curvature of the parameter. The normal to the surface bisects the angle α. Therefore, the impact parameter and the orientation of the surface will determine the general direction of scatter. The radius of curvature will determine the spread of the scattered beam. Therefore, the impact parameter and a complete description of the surface are needed to predict the details of the pattern of scatter. For a complete description of the surface the direction of the normal at some point needs to be specified, then the rest of the perimeter could be constructed using the differential equation $R = R(\theta)$. This type of complete description is not available in the atomic or nuclear case.

From an experimental point of view, the shape of the target must be inferred from the pattern of scatter. The density of projectiles scattered to an angle α determines the radius of curvature of the target at some angle θ. This reconstructed shape will not, in general, match the target's shape because the relationship between α and θ may be very complicated. For example, if part of the surface is concave this analysis will generate a convex shape that is consistant with the data.

For an elliptical target, the two parts of its surface are such that Eqs. (2)–(4) are applicable. The simple relationship, $\alpha = 2\theta$, at the points of maximum and minimum radii of a curvature for an elliptical target make the problem of determining its shape and orientation accessible. For an ellipse the radius of curvature[5] is given by

$$R(\beta) = \{[a \sin(\beta)]^2 + [b \cos(\beta)]^2\}^{1.5}/ab\,, \quad (5)$$

where β is measured internally in the target from its major axis, a is the length of its semimajor axis, and b is the length of the semiminor axis. Let the major axis of the ellipse be at an angle θ_L to the direction of the incoming stream of projectiles then the minor axis will be 90° from θ_L at an angle θ_S. The radius of curvature at the end of the major axis is given by $R_L = b^2/a$ and at the end of the minor axis is $R_S = a^2/b$. Simple algebra gives $a^3 = (R_L R_S^2)$ and $b^3 = (R_L^2 R_S)$.

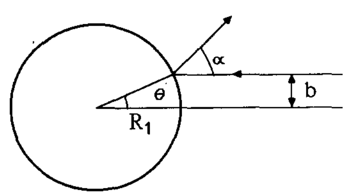

Fig. 1. For a circular target $\alpha = 2\theta$. The coordinates are chosen so α will increase as b increases.

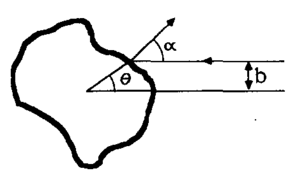

Fig. 3. For an arbitrarily shaped target α is a complicated function of θ.

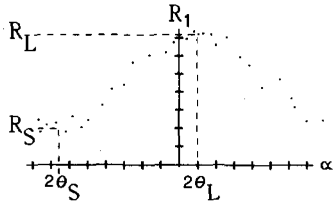

Fig. 4. R_L, R_S, θ_L and θ_S are read directly from R_1 vs α graph.

The normal to the ellipse at the end of its axis extended through the center of the ellipse. Therefore, the angle α will be twice the size of θ when θ is equal to θ_L or to θ_S. A graph of effective radius of curvature versus α will have a maximum of R_S at $\alpha = 2\theta_S$ and a minimum of R_L at $\alpha = 2\theta_L$.

III. EXPERIMENTAL PROCEDURE

Pressure-sensitive paper tape is fixed to the inside of the scattering apparatus shown in Fig. 2. The $-90°$, $0°$, $90°$, and $180°$ values of α are marked on the paper using a carpenter's square. Several ball bearings are fired as the impact parameter is changed in such a way as to produce an isotropic beam of particle. The intensity of the beam is noted with the units of #/inch and given the symbol I. A paper mask, with a rectangular hole (ds) inches wide and as tall as the pressure-sensitive paper, is used to simulate a detector. This detector is moved along the paper and the number of impact points (N_W) showing through the rectangular hole is recorded as the position(s) of the detector's center is varied. With these numbers R_1 is calculated and a graph of R_1 vs α is constructed. A schematic of the graph of data from an elliptical target is shown in Fig. 4. The student is able to determine the appropriate values of R and θ, and thereby specify the dimensions and orientation of the target. The target then is uncovered to confirm the results.

IV. TOWARD A BETTER PEDAGOGY

If the analogy fits, use it. As Prowse points out, much of the analysis of the collision presented in the instructions for the Welch apparatus serves to confuse the discussion with details that are not relevant to the analogy. In addition, since 1961 a concluding statement has been added to these instructions that tends to act as a warning not to proceed with the elliptical targets because the analysis is more complex.

I have found that this material is accessible even to first-year students. The instructor may need to lead them through the theoretical analysis for the circular target. The students appear to readily accept the change of emphasis from the radius of the target to the radius of curvature of the target. The better understanding of how real laboratory measurements are made justifies the extra effort to use the Welch's scattering apparatus properly.

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Improvements in the demonstration of the hysteresis loops of ferromagnetic materials

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An improved apparatus has been developed for demonstrating the hysteresis loops of ferromagnetic materials. Using a Hall probe, a storage oscilloscope, and specially designed circuitry one can in effect plot the initial magnetization curve and the hysteresis loop of a sample point by point, either manually or automatically. Under manual control, the apparatus can also be used to demonstrate minor hysteresis loops of the sample. These minor loops are related to the concepts of permeability μ, incremental permeability μ_Δ and reversible permeability μ_r.

I. INTRODUCTION

The apparatus ordinarily used to demonstrate the hysteresis loops of ferromagnetic materials[1] has a number of inherent limitations. In particular the 50–60 Hz ac is too high a frequency to allow the eye to follow the spot on the CRT screen as it traces out the hysteresis loop. Neither can one, with this apparatus, trace out the initial magnetization